String Waves

How they can be used to understand seismic waves

Part 1: Defining The Terms

- **Seismology** is the study of Earthquakes (tectonic plates, tsunamis, volcanic eruptions, etc), and everything related to them.
- Earthquake-related phenomena is studied by studying **Seismic Waves.** Seismic Waves are caused by any sudden movement of the Earth, and by analyzing them, seismologists are able to learn more about Earthquakes and the Earth’s interior; its structure.

But seismic waves are very complicated, which begs the question: how can we make it easier to properly understand them? Well, a good way would be to analogize them to **the waves in a 1D vibrating string.**

Part 1 Cont

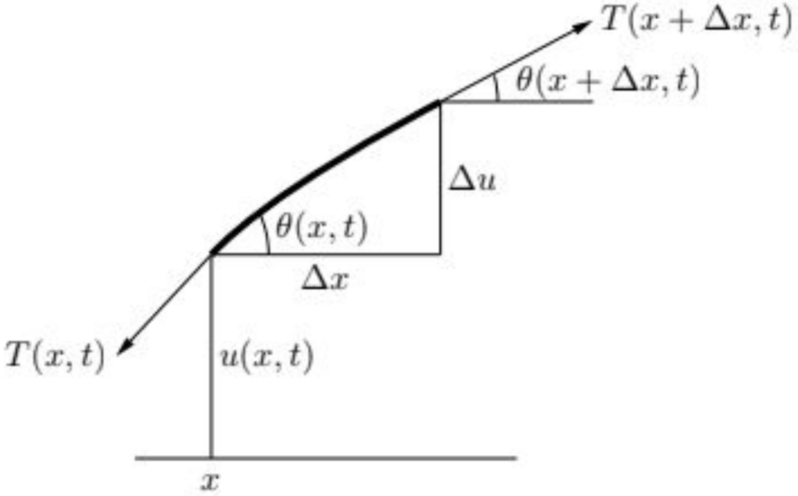
- Waves are perhaps best understood when analyzing them within the context of **Violin Strings and its physics.** Think about what happens a violin string is plucked; it makes a sound, and the sound varies depending on seemingly insignificant nuances with regards to how and where you pull. Well, this phenomenon can be explained by thinking of the strings as…waves. On a violin string, both ends are stationary - enabling the presence of a standing wave. These waves are then perceived by us humans as sound. The Earth is the same way in a sense; when there is an earthquake, a seismic wave vibration is generated by the Earth.

Part 1 Cont

- **Modes of Vibration:** There are several different ways/patterns of vibrating…and that is what the modes of vibration represents. Typically, oscillation can only occur in a few, distinct patterns: symmetric stretching, asymmetric stretching, wagging, twisting, scissoring, and rocking. In short, they are stationary waves that oscillate with the same pattern.

What is the Wave Equation?

To put into context how the Earth moves mathematically, we need the wave equation (which is what will be represented via the coding - more on that in just a bit).



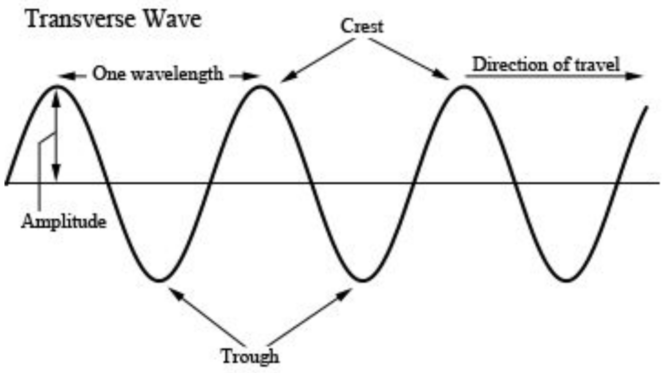
Wavelength = Exactly what it says, the distance of a wave; between two crests.

Wavenumber = Measurement of spatial frequency

Frequency = The number of waves that pass a given point in 1 second

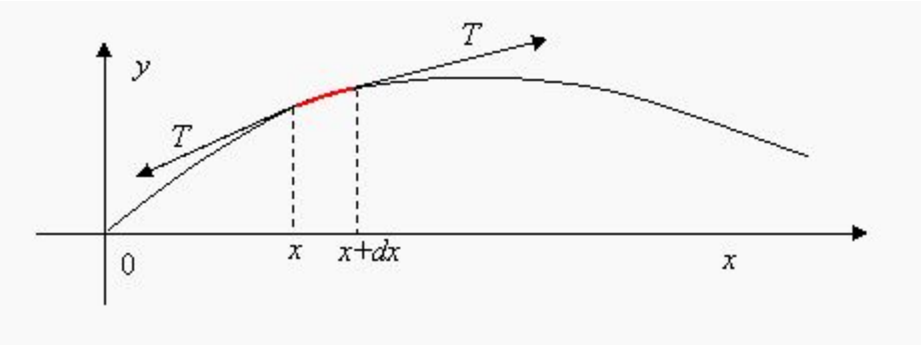
Derivation of Wave Equation

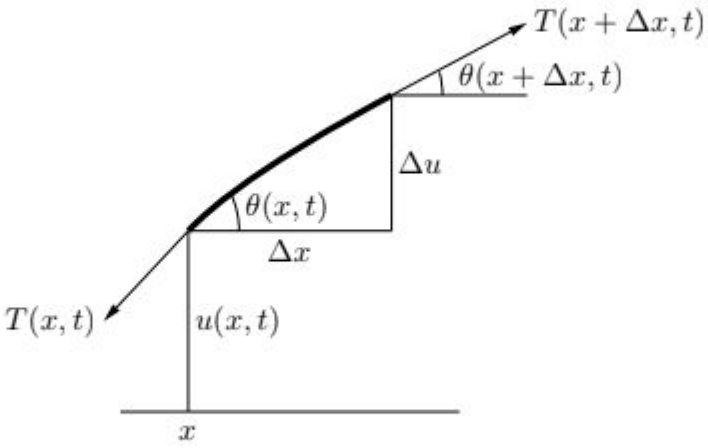
- First, to make this as visually comprehensible as possible, let's draw a 1D string on a graph, setting the x and y axes at L and U respectively.
- The immediate mathematical conclusion from the above is that vertical displacement of the string (u) is a function of t (time) and x (the horizontal displacement). So, u as a function of t,x measures the displacement of the string vertically at a given time and spatial location.
- We need to know more than that, though. How can we use the above equation to derive a final equation that measures how the string moves?



Derivation of Wave Equation Cont

- To do that, we'd have to pinpoint a very small part of the 1D string, tailored enough to where we can essentially view the curve as a slope. To make things more complete and clarified, we'd also have to add the tension and the angles.





Derivation of Wave Equation: The Final Result

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

$$v = \sqrt{\frac{T}{\rho}}$$

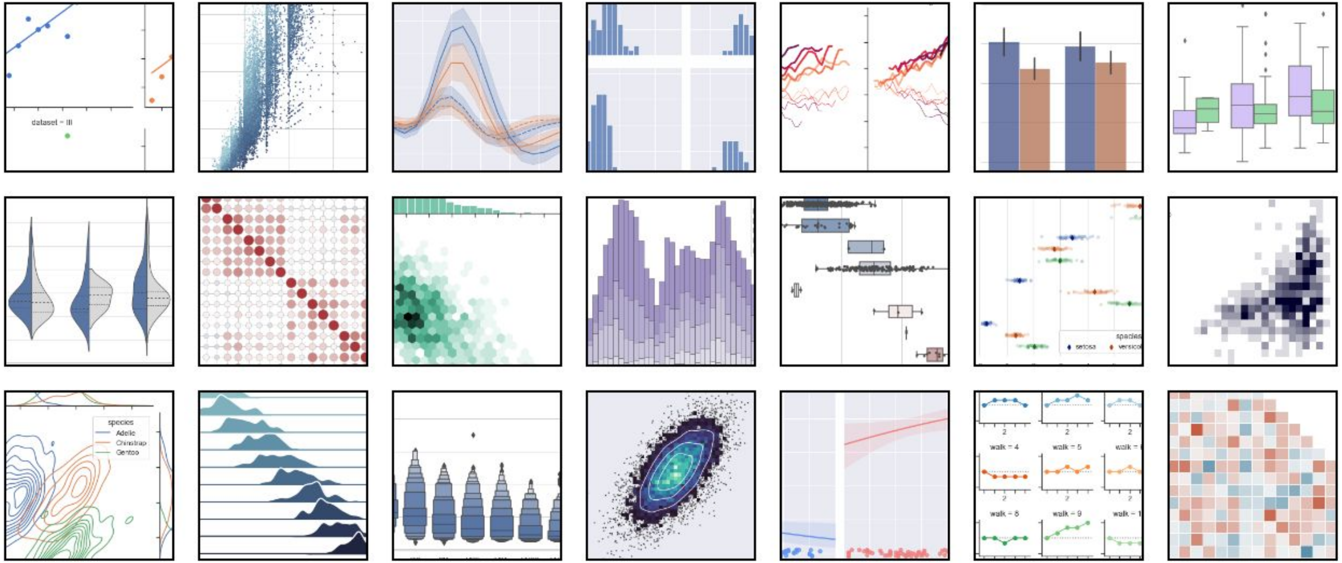
Wave Equation (velocity on the right): differential equation describing wave propagation (for string). Vertical acceleration = velocity^ x curvature.

$$u_n(x,t) = \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right), \quad \text{where } n = 1, 2, 3...$$

Solution when both ends of the string are fixed

Part 2: The Coding

- Using the aforementioned knowledge, the goal was to create graphs and animations on matplotlib to illustrate the wave equations as functions, and therefore the movements of the Earth. So, how does the code work?



Part 2 Cont

First, it is important to establish certain aliases with the program - numpy as np, matplotlib.pyplot as plt. This of course makes things a lot more efficient.

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from matplotlib import rc
```

Next, it is time to define each and every one of the variables. This is especially important, as without doing so, there is no way for the program to generate graphs using the ensuing equations.

```
N = 500
L = 1.0
x = np.linspace(0, L, N)
t = np.linspace(0, 2, 200)
c = 0.5
```

Now that all of the important variables are clearly defined, we can enter the equations we want to illustrate. The top column defines the function, and its variable (above). The second column is the actual equation. Note the usage of numpy - that is how the code understands “sin”, “cos”, and “pi”.

```
def u_n(x, t, n, L, c):
  u = np.sin((n*np.pi*x)/L)*np.cos((n*np.pi*c*t)/L)
  return u
```

Part 2 Cont

- Now, it’s actually time to plot the figure. To do that, first, you have to compute just that: create the figure. Plot the figure. In brackets are the parameters.
- The next line is important for ensuring that multiple graphs will be plotted. Why is this important? Because of the aforementioned modes; more than one mode needs to be represented.
- After that, it is mostly details that serve as the finishing pieces of the puzzle.

```
# Create figure
fig = plt.figure(figsize=(10, 8))

lines = []
for n in [1, 2, 3, 4]:
  plt.subplot(2, 2, n)
  line, = plt.plot(x, u_n(x, t[0], n, L, c), linewidth = 2.0, color = "blue", label = "u")
  lines.append(line)
  plt.xlabel("x")
  plt.ylabel(f"Mode{n}")
  plt.xlim (0.0, L)
  plt.ylim (-1.1, 1.1)
  plt.tick_params(axis='both', which='major', labelsize=10)
  plt.tick_params(axis='both', which='minor', labelsize=18)

# Animation function
def animate(i):
  for n in [0, 1, 2, 3]:
      lines[n].set_ydata(u_n(x, t[i], n + 1, L, c))
  return lines

# Create animation
ani = FuncAnimation(fig, animate, frames=len(t), interval=50, blit=True)

# Display animation
plt.tight_layout()
ani
```

So what does this all show?

The graphs tell us how the displacement of a string can change as time does, so, in essence, it provides a visual of seismic waves. Remember, the waves in the strings were meant as seismic wave equivalents, so the movement of the string - when the wave equation is applied to it - indicate the theoretical movement of seismic waves. The important thing to focus on is the CHANGE, and that’s why the animations are essential here.